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(Printed Pages 4)

Roll No. _____

19/1562

B.C.A. Examination, 2019

Second Semester

Fifth Paper

(Mathematics - II)

Time : Three Hours *Maximum Marks : 75*

Note: Attempt any **five** questions. **All** questions carry equal marks.

The answers to short answer type questions should not exceed 200 words and the answers to long answer type questions should not exceed 500 words.

1. (a) Define finite and infinite sets. Prove that for sets A and B

$(A \cap B)' = A' \cup B'$

i.e. complement of intersection of A and B is the union of complements of A and B. 7½

P.T.O.

(b) If A = {1, 3, 5} and B = {2, 4, 6}, Then calculate A x B and B x A. 7½

2. (a) What do you understand by an equivalence relation on a non empty set A. Let R_0 be the relation on the set R of real numbers given by aR_0b iff $|a-b| < \frac{1}{2}$.

Prove that R_0 is not an equivalence relation. 7½

(b) (i) Let A = {X : −1 < < 1} = B. Let f : A → B be given as $f(x) = \frac{1}{2}x$, ∀ x ∈ A. Is f a one-one, onto function? Give reasons. Also draw the graph of the function. 4

(ii) Prove that the function $f: R \longrightarrow R$ defined by f(x) = 4x + 3 is invertible. Find $f^{-1}: R \longrightarrow R$. 3½

3. Define a partially ordered set (POSET) Let A = {1, 2, 3, 4, 6, 8, 9, 12, 18, 24} be ordered by the relation "a divides b". Draw the Hasse diagram of (A, ≤), where a ≤ b ⇔ a|b ie. a divides b. 15

4. (a) Define maximal and minimal elements in a POSET with examples. Let A = {1, 2, 3, 4, 5, 6} be ordered as pictured in the figure:

Find the upper and lower bounds of the subset B = {4, 5}, of A. 7½

(b) Let S be any non empty set and P(S) be its power set. Show that (P(S) ⊆) is a Lattice. 7½

5. (a) If $u = f(x/y)$, then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0.$$ 7½

(b) Find the extreme value of the function f $(x, y) = x^3 + y^3 - 6(x^2 + y^2) + 12xy - 75(x + y)$. 7½

6. (a) Prove that the lines -

$$\frac{x+3}{2} = \frac{y+5}{3} = \frac{z-7}{-3} \text{ and } \frac{x+1}{4} = \frac{y+1}{5} = \frac{z+1}{-1}$$

are coplaner. Find the equation of the plane containing them. 7½

(b) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and find the point of contact. 7½

7. (a) Evaluate $\iint_R x^2y\,dx\,dy$ over R, where R : $\{0 \le x \le 1 ; 0 \le y \le 2\}$. 7½

(b) Find the whole volume bounded by the surface $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^4}{c^4} = 1$. 7½