20/1085
B.C.A. Examination, 2020
(First Semester)
Fifth Paper
Mathematics - I

Time : Three Hours ***Maximum Marks : 75***

Note: Answer any **five** questions. **All** questions carry equal marks.

Note : The answers to short answer type questions should not exceed 200 words and the answers to long answer type questions should not exceed 500 words.

P.T.O.

1. (a) Find the inverse of the matrix 7½

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

(b) Find the rank of the matrix 7½

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$$

2. (a) Examine the continuity of the function f(x) at x = 0

$$f(x) = \frac{x e^{1/x}}{1 + e^{1/x}}, \quad x \neq 0$$

$$= 0, \quad x = 0.$$ 7½

(b) Find $\lim_{x \to 0} \frac{\log x}{\cot x}$. 7½

3. (a) Find the differential coefficient of

$$(\sin x)^{\log x}$$ 7½

(b) Is the function f(x) = |x| differentiable at x = 0? 7½

4. (a) If $y = a \cos(\log x) + b \sin(\log x)$

show that $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$. 7½

(b) If $y^{1/m} + y^{-1/m} = 2x$ 7½

Prove that :

$$(x^2-1)y_{n+2}+(2n+1)xy_{n+1} - (n^2-m^2)y_n=0$$

5. (a) Expand sin x by Maclaurins theorem. 7½

(b) State and prove Lagrange's mean value theorem. 7½

6. (a) Evaluate $\int \frac{dx}{x(x^4-1)}$. 7½

(b) Evaluate $\int \cos^7 x \, dx$. 7½

7. (a) Show that

$[\bar{a} \times \bar{b}, \bar{b} \times \bar{c}, \bar{c} \times \bar{a}] = [\bar{a}\,\bar{b}\,\bar{c}]^2$ 7½

(b) Find the value of a such that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + a\hat{j} + 5\hat{k}$ are coplanar. 7½

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8. (a) Find the area of the parallelogram determined by the vectors

$\hat{i} + 2\hat{j} + 2\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$. 7½

(b) Show that

$$(\bar{a} - \bar{b}) \times (\bar{a} + \bar{b}) = 2\bar{a} \times \bar{b}$$

and give its geometrical interpretation. 7½

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