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(Printed Pages 4)

Roll No. _____

19/185

B.C.A. (First Semester)

Examination, 2019

Fifth Paper

(Mathematics - I)

Time : Three Hours Maximum Marks : 75

Note: Answer any **five** questions. **All** questions carry equal marks. The answers to short type questions should not exceed 200 words and answers to long answer type questions should not exceed 500 words.

1. (a) Find the rank of the martrix. 7½

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ -1 & -3 & -4 & -3 \end{bmatrix}$$

(b) For the two matrices 7½

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

Compute AB and BA and show that $AB \neq BA$

2. (a) Examine for continuity the function 7½

$$f(x) = \frac{e^{1/x^2}}{e^{1/x^2} - 1}, \quad x \neq 0$$

$$= 1, \quad x = 0$$

at $x = 0$

(b) Evaluate : 7½

$$\lim_{x \to 0} \frac{\sin x}{x}$$

3. (a) Examine the function for continuity at $x=0$, 7½

$f(x) = \frac{\sin^2 ax}{x^2}$ for $x \neq 0$, $f(x) = 1$ for $x=0$

(b) Let f(x) be an even function. If f'(0) exists, find its value. 7½

4. (a) State and prove Maclaurin's theorem. 8

(b) Find the limit 7

$$\lim_{x\to 0}\frac{x-\sin x}{x^3}$$

5. (a) If $y = e^{a\sin^{-1}x}$ 8

Prove that

$$(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$$

(b) Find the maximum value of 7

$$(x-1)(x-2)(x-3)$$

6. (a) Evaluate : $\int\frac{x^2+x+2}{(x-2)(x-1)}dx$ 7

(b) Use reduction formula to integrate

$$\int \sin^4 x\cos^5 x\,dx$$ 8

7. (a) Prove that 7½

$$\hat{i}\times(\bar{a}\times\hat{i})+\hat{j}\times(\bar{a}\times\hat{j})+\hat{k}\times(\bar{a}\times\hat{k})=2\bar{a}$$

(b) Prove that 7½

$$\left[\bar{a}+\bar{b}\quad b+\bar{c}\quad \bar{c}+\bar{a}\right]=2\left[\bar{a}\bar{b}\bar{c}\right]$$

8. Explain with examples : 7+8

(a) Scalar product of Two Vectors and its geometric significance.

(b) Vector product of Two vectors and its geometric significance.