B.C.A study

Unit-1:Sets

Set Theory

It is natural for us to classify items into groups, or sets, and consider how those sets overlap with each other. We can use these sets understand relationships between groups, and to analyze survey data.

Basics

An art collector might own a collection of paintings, while a music lover might keep a collection of CDs. Any collection of items can form a **set**.

SET

A **set** is a collection of distinct objects, called **elements** of the set

A set can be defined by describing the contents, or by listing the elements of the set, enclosed in curly brackets.

EXAMPLE 1

Some examples of sets defined by describing the contents:

1. The set of all even numbers

2. The set of all books written about travel to Chile

Answers

Some examples of sets defined by listing the elements of the set:

1. {1, 3, 9, 12}
2. {red, orange, yellow, green, blue, indigo, purple}

A set simply specifies the contents; order is not important. The set represented by {1, 2, 3} is equivalent to the set {3, 1, 2}.

NOTATION

Commonly, we will use a variable to represent a set, to make it easier to refer to that set later.

The symbol $\in$ means "is an element of".

A set that contains no elements, { }, is called the **empty set** and is notated $\emptyset$

EXAMPLE 2

Let $A = \{1, 2, 3, 4\}$

To notate that 2 is element of the set, we'd write $2 \in A$

Sometimes a collection might not contain all the elements of a set. For example, Chris owns three Madonna albums. While Chris's collection is a set, we can also say it is a **subset** of the larger set of all Madonna albums.

SUBSET

A **subset** of a set A is another set that contains only elements from the set A, but may not contain all the elements of A.

If B is a subset of A, we write $B \subseteq A$

A **proper subset** is a subset that is not identical to the original set—it contains fewer elements.

If B is a proper subset of A, we write $B \subset A$

EXAMPLE 3

Consider these three sets:

A = the set of all even numbers
$B = \{2, 4, 6\}$
$C = \{2, 3, 4, 6\}$

Here $B \subset A$ since every element of B is also an even number, so is an element of A.

More formally, we could say $B \subset A$ since if $x \in B$, then $x \in A$.

It is also true that $B \subset C$.

C is not a subset of A, since C contains an element, 3, that is not contained in A

EXAMPLE 4

Suppose a set contains the plays "Much Ado About Nothing," "MacBeth," and "A Midsummer's Night Dream." What is a larger set this might be a subset of?

There are many possible answers here. One would be the set of plays by Shakespeare. This is also a subset of the set of all plays ever written. It is also a subset of all British literature.

Union, Intersection, and Complement

Commonly sets interact. For example, you and a new roommate decide to have a house party, and you both invite your circle of friends. At this party, two sets are being combined, though it might turn out that there are some friends that were in both sets.

UNION, INTERSECTION, AND COMPLEMENT

The **union** of two sets contains all the elements contained in either set (or both sets). The union is notated $A \cup B$. More formally, $x \in A \cup B$ if $x \in A$ or $x \in B$ (or both)

The **intersection** of two sets contains only the elements that are in both sets. The intersection is notated $A \cap B$. More formally, $x \in A \cap B$ if $x \in A$ and $x \in B$.

The **complement** of a set A contains everything that is *not* in the set A. The complement is notated A', or A^c, *or sometimes* $\sim A$.

EXAMPLE 5

Consider the sets:

A = {red, green, blue}
B = {red, yellow, orange}
C = {red, orange, yellow, green, blue, purple}

Find the following:

1. Find $A \cup B$
2. Find $A \cap B$
3. Find $A^c \cap C$

Answers

1. The union contains all the elements in either set: $A \cup B$ = {red, green, blue, yellow, orange} Notice we only list red once.
2. The intersection contains all the elements in both sets: $A \cap B$ = {red}
3. Here we're looking for all the elements that are *not* in set A and are also in C. $A^c \cap C$ = {orange, yellow, purple}

UNIVERSAL SET

A **universal set** is a set that contains all the elements we are interested in. This would have to be defined by the context.

A complement is relative to the universal set, so A^ccontains all the elements in the universal set that are not in A.

EXAMPLE 6

1. If we were discussing searching for books, the universal set might be all the books in the library.
2. If we were grouping your Facebook friends, the universal set would be all your Facebook friends.
3. If you were working with sets of numbers, the universal set might be all whole numbers, all integers, or all real numbers

EXAMPLE 7

Suppose the universal set is U = all whole numbers from 1 to 9. If A = {1, 2, 4}, then A^C = {3, 5, 6, 7, 8, 9}.

Cardinality

Often times we are interested in the number of items in a set or subset. This is called the cardinality of the set.

CARDINALITY

The number of elements in a set is the cardinality of that set.

The cardinality of the set A is often notated as $|A|$ or n(A)

CARDINALITY PROPERTIES

n($A \cup B$) = n(A) + n(B) – n($A \cap B$)

n(Ac) = n(U) – n(A)

EXAMPLE 12

Let A = {1, 2, 3, 4, 5, 6} and B = {2, 4, 6, 8}.

What is the cardinality of B? $A \cup B$, $A \cap B$?

Answers

The cardinality of B is 4, since there are 4 elements in the set.

The cardinality of $A \cup B$ is 7, since $A \cup B$ = {1, 2, 3, 4, 5, 6, 8}, which contains 7 elements.

The cardinality of $A \cap B$ is 3, since $A \cap B$ = {2, 4, 6}, which contains 3 elements.

EXAMPLE 13

What is the cardinality of P = the set of English names for the months of the year?

Answers

The cardinality of this set is 12, since there are 12 months in the year.

Cartesian product definition

For two sets *A* and *B*, the Cartesian product of *A* and *B* is denoted by *A*×*B* and defined as:

```
A×B = { (a,b) | aϵA and bϵB }
```

Cartesian Product is the multiplication of two sets to form the set of all ordered pairs. The first element of the ordered pair belong to first set and second pair belong the second set. For an example,

```
Suppose, A = {dog, cat}
         B = {meat, milk}  then,
         A×B = {(dog,meat), (cat,milk), (dog,milk), (cat,meat)}
```

Cartesian Product Examples

Example #1: Practical Example on Cartesian Product

If *X* be the set of points on x-plane and *Y* be the set of points on y-plane then, *X*×*Y* represents the on *XY* plane.

Example #2

Suppose two sets A = {a, b} and B = {1, 2, 3}. Find A×B and B×A.

```
Here,
A = {a, b}
B = {1, 2, 3}
Now,
A×B = {(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)}
B×A = {(1,a), (1,b), (2,a),(2,b), (3,a), (3,b)}
```

Cartesian product of two sets

Example #3

If A×B = { (x,1), (x,2), (x,3),(y,1), (y,2), (y,3)}. Find A and B.

Since, set A contains first element of each ordered pair only,

```
A = {x, y}
```

Since, set B contain second element of ordered pair only,

```
B = {1, 2, 3}
```

We exclude duplicate elements in both sets (because, set can only contain unique element).

Cartesian Product of 3 Sets

For three sets A, B and C, the Cartesian product of A, B and C is denoted by A×B×C and defined as:

A×B×C = { (p, q, r) | p∊A and q∊B and r∊C }

Key Points on Cartesian Product

Cartesian Product of Empty Set

If either of two set is empty, the Cartesian product of those two set is also an empty.

If $A = \{1, 2\}$ and $B = \phi$. Then, $A \times B = \phi$ and $B \times A = \phi$.

Non-commutativity Property

For two unique and non-empty sets A and B, $A \times B$ is not equal to $B \times A$.

If $A = \{a, b\}$ and $B = \{1, 2, 3\}$ then,

```
A×B = {(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)}
B×A = {(1,a), (1,b), (2,a),(2,b), (3,a), (3,b)}

Here, A×B ≠ B×A
```

Condition for Commutative Property

For two sets A and B, the Cartesian product of two sets A×B and B×A are equal if either of the following condition is satisfied:

- either of two set is empty
- both sets are equal

If $A = \{1, 2\}$ and $B = \phi$. Then,

```
          A×B = ϕ
          B×A = ϕ
    Hence, A×B = B×A
```

If $A = B = \{1, 2\}$ then,

$A \times B = \{(1,2), (2,2), (2,1), (2,2)\}$
$B \times A = \{(1,2), (2,2), (2,1), (2,2)\}$
$A \times A = A^2 = \{(1,2), (2,2), (2,1), (2,2)\}$
$B \times B = B^2 = \{(1,2), (2,2), (2,1), (2,2)\}$

Hence, $A \times B = B \times A = A^2 = B^2$

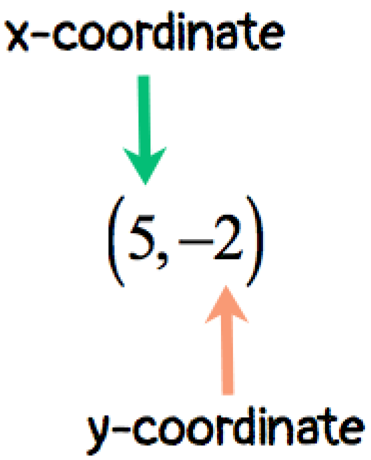
...

B.C.A study

Unit -2: Relation and Function

Let's start by saying that a **relation** is simply a set or collection of ordered pairs. Nothing really special about it. An ordered pair, commonly known as a point, has two components which are the x and y coordinates.

This is an example of an ordered pair.



Main Ideas and Ways How to Write or Represent Relations

As long as the numbers come in pairs, then that becomes a relation. If you can write a bunch of points (ordered pairs) then you already know how a relation looks like. For instance, here we have a relation that has five ordered pairs. Writing this in set notation using curly braces.

$$\{(-2,1),(-2,3),(0,-3),(1,4),(3,1)\}$$

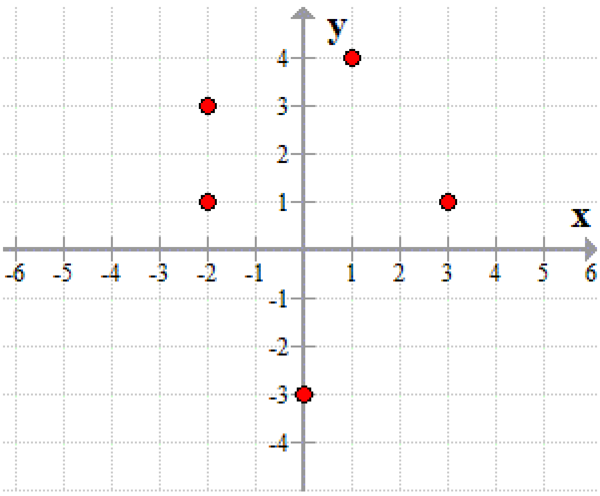
Relation in set notation:

However, aside from set notation, there are other ways to write this same relation. We can show it in a table, plot it on the xy-axis, and express it using a mapping diagram.

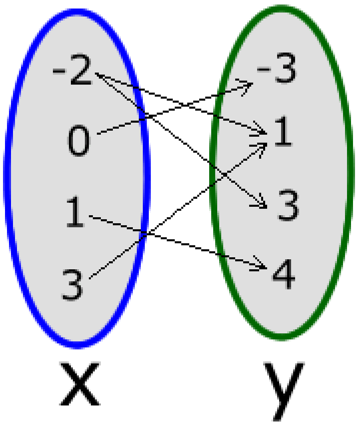
- **Relation in table**

x	y
-2	1
-2	3
0	-3
1	4
3	1

- **Relation in graph**



- **Relation in mapping diagram**



- The **domain** is the set of all x or input values. We may describe it as the collection of the *first values* in the ordered pairs.
- The **range** is the set of all y or output values. We may describe it as the collection of the *second values* in the ordered pairs.

So then in the relation below

$$\{(-2,1),(-2,3),(0,-3),(1,4),(3,1)\}$$

our domain and range are as follows:

Domain	Range
$\{-2,0,1,3\}$	$\{-3,1,3,4\}$

When listing the elements of both domain and range, get rid of duplicates and write them in increasing order.

Relations and Functions

Let's start by saying that a **relation** is simply a set or collection of ordered pairs. Nothing really special about it. An ordered pair, commonly known as a point, has two components which are the x and y coordinates.

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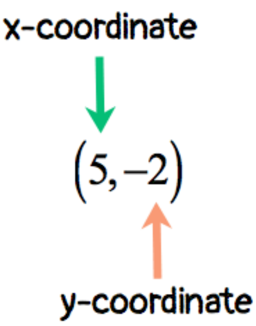

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Relation in set notation:

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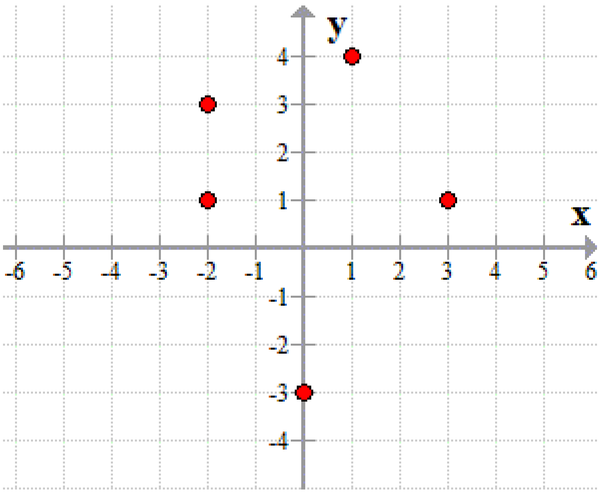
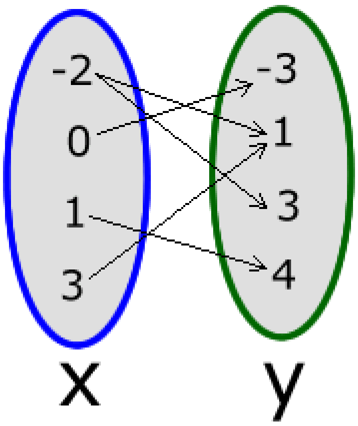


- **Relation in mapping diagram**



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So then in the relation below

$\{(-2,1), (-2,3), (0,-3), (1,4), (3,1)\}$

our domain and range are as follows:

Domain	Range
$\{-2, 0, 1, 3\}$	$\{-3, 1, 3, 4\}$

When listing the elements of both domain and range, get rid of duplicates and write them in increasing order.

What Makes a Relation a Function?

On the other hand, a **function** is actually a "special" kind of relation because it follows an extra rule. Just like a relation, a function is also a set of ordered pairs; however, every x-value must be associated to only one y-value.

Suppose we have two relations written in tables,

- A relation that is **not a function**

x	y
-3	7
-1	5
0	-2
5	9
5	3

Since we have repetitions or duplicates of x-values with different y-values, then this relation ceases to be a function.

- A relation that is **a function**

x	y
-2	0
-1	-2
0	3
4	-1
5	-3

This relation is definitely a function because every x-value is unique and is associated with only one value of y.

Relations and Functions

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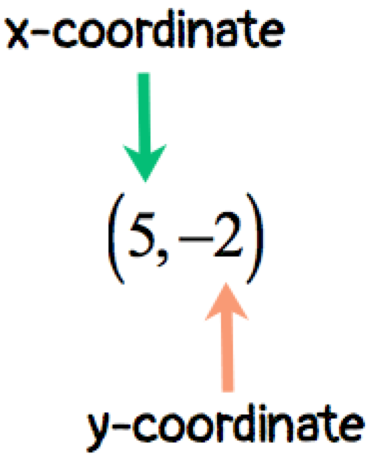

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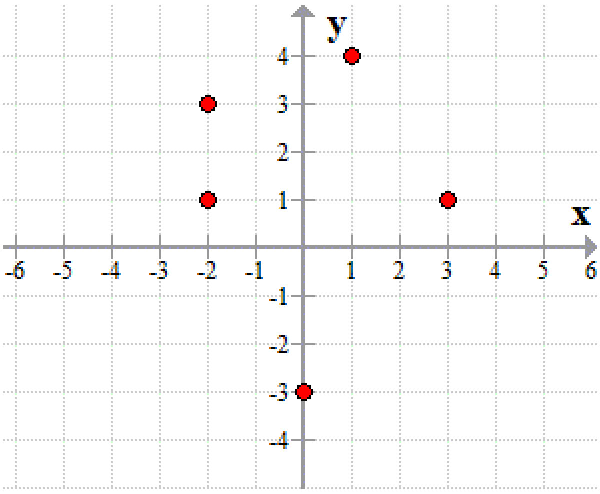
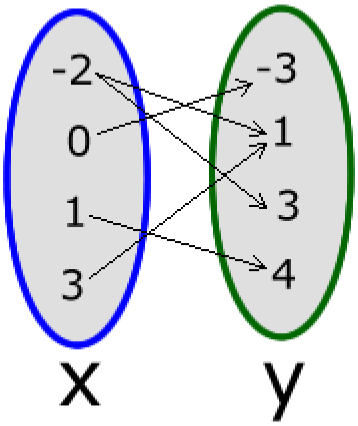


- **Relation in mapping diagram**



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So then in the relation below

$$\{(-2,1), (-2,3), (0,-3), (1,4), (3,1)\}$$

our domain and range are as follows:

Domain	Range
$\{-2, 0, 1, 3\}$	$\{-3, 1, 3, 4\}$

When listing the elements of both domain and range, get rid of duplicates and write them in increasing order.

What Makes a Relation a Function?

On the other hand, a **function** is actually a “special” kind of relation because it follows an extra rule. Just like a relation, a function is also a set of ordered pairs; however, every x-value must be associated to only one y-value.**https://tpc.googlesyndication.com/safeframe/1-0-37/html/container.htm (https://tpc.googlesyndication.com/safeframe/1-0-37/html/container.htm)**

Suppose we have two relations written in tables,

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-3	7
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5	9
5	3

Since we have repetitions or duplicates of x-values with different y-values, then this relation ceases to be a function.

- A relation that is **a function**

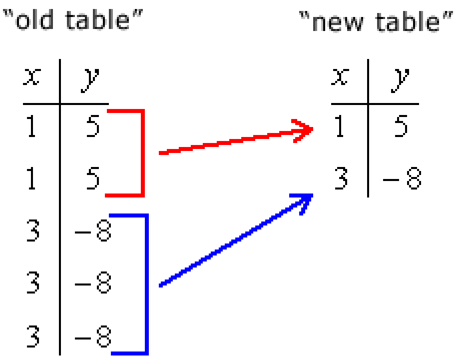
x	y
-2	0
-1	-2
0	3
4	-1
5	-3

This relation is definitely a function because every x-value is unique and is associated with only one value of y.

So for a quick summary, if you see any duplicates or repetitions in the x-values, the relation is not a function. How about this example though? Is this not a function because we have repeating entries in x?

x	y
1	5
1	5
3	-8
3	-8
3	-8

Be very careful here. Yes, we have repeating values of x but they are being associated with the same value of y. The point (1,5) shows up twice, and while the point (3,-8) is written three times. This table can be cleaned up by writing a single copy of the repeating ordered pairs.

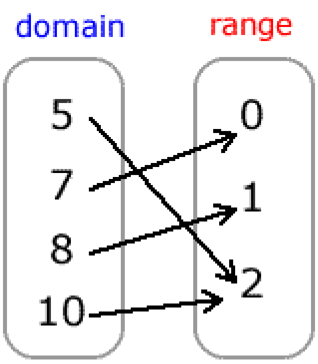


The relation is now clearly a function!

Examples of How to Determine if a Relation is also a Function

Let's go over a few more examples by identifying if a given relation is a function or not.

Example 1: Is the relation expressed in the mapping diagram a function?



Each element of the domain is being traced to one and only element in the range. However, it is okay for two or more values in the domain to share a common value in the range. That is, even though the elements 5 and 10 in the domain share the same value of 2 in the range, this relation is still a function.null

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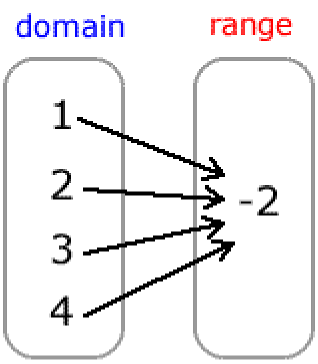
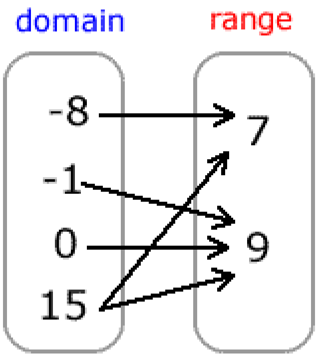
Example 2: Is the relation expressed in the mapping diagram a function?



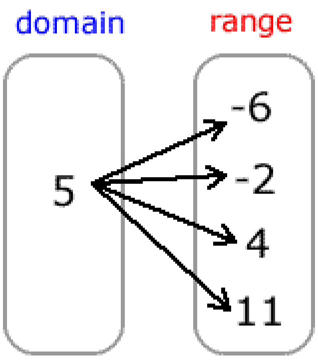
What do you think? Does each value in the domain point to a single value in the range? Absolutely! There's nothing wrong when four elements coming from the domain are sharing a common value in the range. This is a great example of a function as well.

Example 3: Is the relation expressed in the mapping diagram a function?



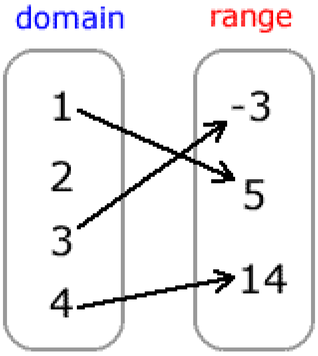
Messy? Yes! Confusing? Not really. The only thing I am after is to observe if an element in the domain is being "greedy" by wanting to be paired with more than one element in the range. The element 15 has two arrows pointing to both 7 and 9. This is a clear violation of the requirement to be a function. A function is well behaved, that is, each element in the domain must point to one element in the range. Therefore, this relation is **not a function**.

Example 4: Is the relation expressed in the mapping diagram a function?



If you think example 3 was "bad", this is "worse". A single element in the domain is being paired with four elements in the range. Remember, if an element in the domain is being associated with more than one element in the range, the relation is automatically disqualified to be a function. Thus, this relation is absolutely **not a function**.

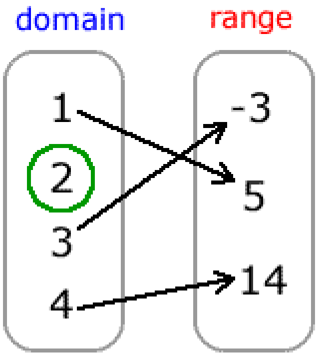
Example 5: Is the mapping diagram a relation, or function?



Let me show you this example to highlight a very important idea about a function that is usually ignored. Your teacher may give you something like this just to check if you pay attention to the details of the definition of a function.null

So far it looks normal. But there's a little problem. The element "2" in the domain is not being paired with any element in the range.

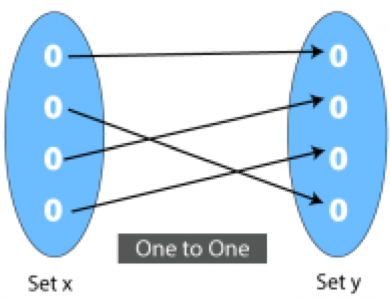
Here's the deal! Every element in the domain **must** have some kind of correspondence to the elements in the range for it to be considered a relation, at least. Since this is not a relation, it follows that it can't be a function.



So, the final answer is **neither** a relation nor a function.

Definition of One-to-One Functions

A function has many types and one of the most common functions used is the **one-to-one function or injective function.** Also, we will be learning here the inverse of this function.



One-to-One functions define that each element of one set say Set (A) is mapped with a unique element of another set, say, Set (B).

Or

Itcould be defined as each element of Set A has a unique element on Set B.

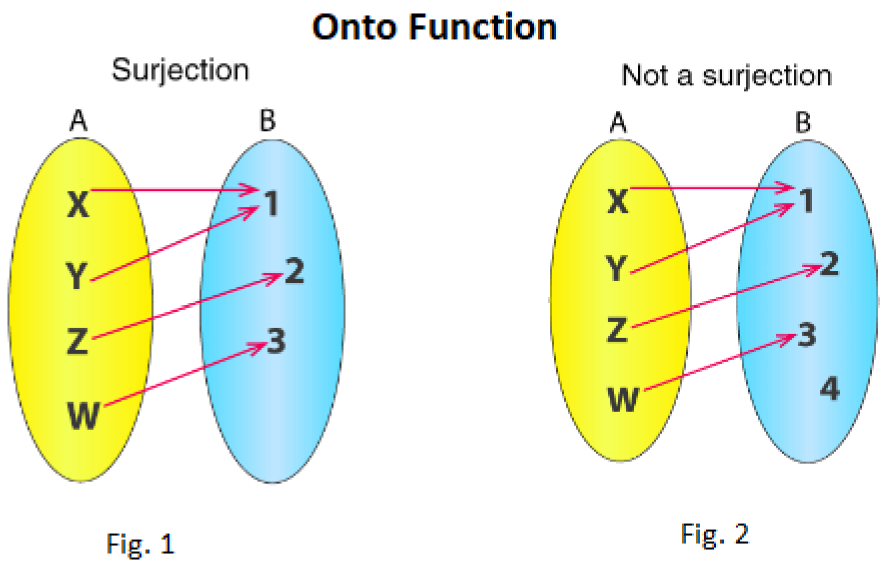
In brief, let us consider 'f' is a function whose domain is set A. The function is said to be injective if for all x and y in A,

Whenever f(x)=f(y), then x=y

And equivalently, if x ≠ y, then f(x) ≠ f(y)

Onto Function Definition (Surjective Function)

Onto function could be explained by considering two sets, Set A and Set B which consist of elements. If for every element of B there is at least one or more than one element matching with A, then the function is said to be **onto function** or surjective function. The term for the surjective function was introduced by Nicolas Bourbaki.



Onto Function

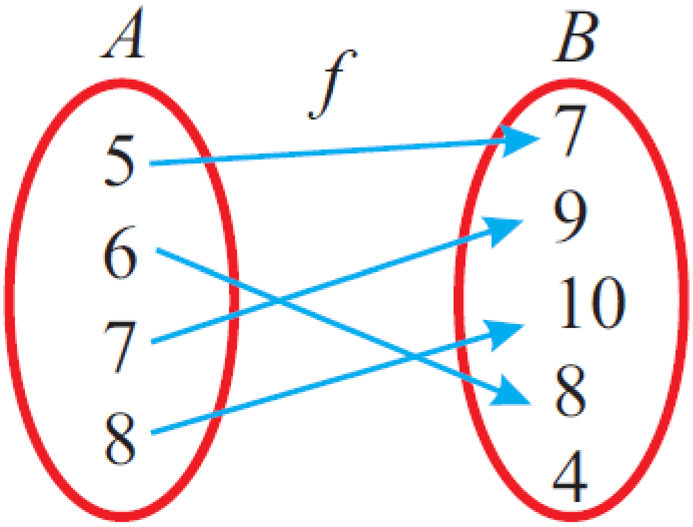
In the first figure, you can see that for each element of B there is a pre-image or a matching element in Set A, therefore, its an onto function. But if you see in the second figure, one element in Set B is not mapped with any element of set A, so it's not an onto or surjective function.

Into Function :

Let f : A —-> B be a function.

There exists even a single element in B having no pre-image in A, then f is said to be an into function.

The figure given below represents a one-one function.



Into Function – Practice Problems

Problem 1 :

Let f : A —-> B. A, B and f are defined as

A = {1, 2, 3}

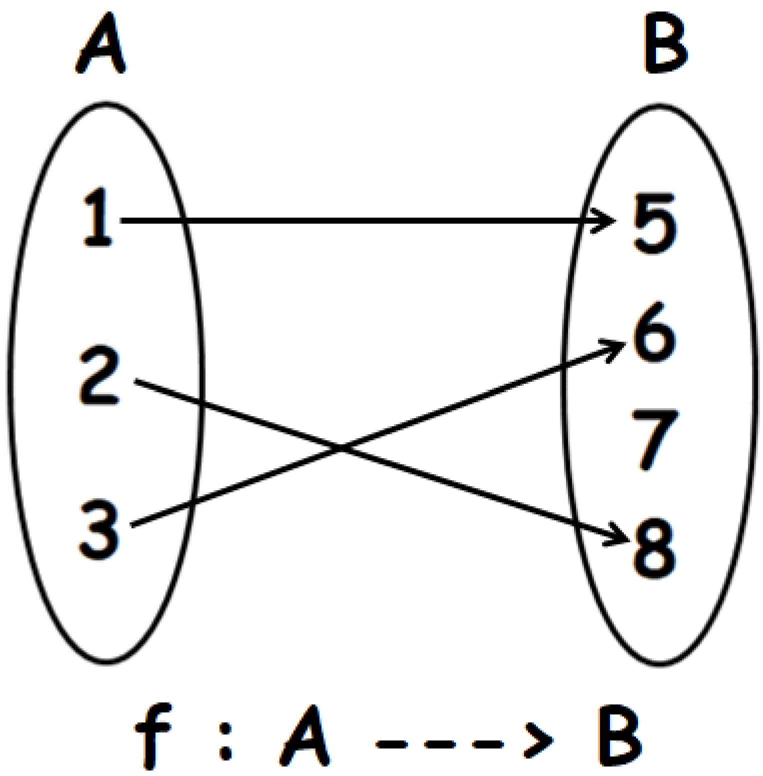
B = {5, 6, 7, 8}

f = {(1, 5), (2, 8), (3, 6)}

Is f into function? Explain.

Solution :

Write the elements of f (ordered pairs) using arrow diagram as shown below



In the above arrow diagram, all the elements of A have images in B and every element of A has a unique image.

That is, no element of A has more than one image.

So, f is a function.

There exists an element "7" in B having no pre-image in A.

Therefore, f is into function.

Introduction to trigonometry

Trigonometry is primarily a branch of mathematics that deals with triangles, mostly right triangles. In particular the ratios and relationships between the triangle's sides and angles.

Trigonometry functions – introduction

There are six functions that are the core of trigonometry. There are three primary ones that you need to understand completely:

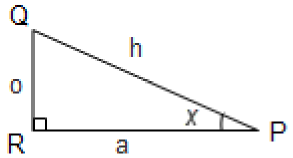
- Sine (sin)
- Cosine (cos)
- Tangent (tan)

The other three are not used as often and can be derived from the three primary functions. Because they can easily be derived, calculators and spreadsheets do not usually have them.

- Secant (sec)
- Cosecant (csc)
- Cotangent (cot)

All six functions have three-letter abbreviations (shown in parentheses above).

Definitions of the six functions



Consider the right triangle above. For each angle P or Q, there are six functions, each function is the ratio of two sides of the triangle. **The only difference between the six functions is which pair of sides we use.**

In the following table

- **a** is the length of the side **a**djacent to the angle (x) in question.
- **o** is the length of the side **o**pposite the angle.
- **h** is the length of the Hypotenuse**. (https://www.mathopenref.com/hypotenuse.html)**

"x" represents the measure of ther angle in either degrees or radians.

Sine	$\sin x = \frac{o}{h}$	The three primary functions
Cosine	$\cos x = \frac{a}{h}$	
Tangent	$\tan x = \frac{o}{a}$	

In the following table, note how each function is the reciprocal of one of the basic functions sin, cos, tan

	Definition	**Reciprocal**
Cosecant	$\csc x = \frac{h}{o}$	$\csc x = \frac{1}{\sin x}$
Secant	$\sec x = \frac{h}{a}$	$\sec x = \frac{1}{\cos x}$
Cotangent	$\cot x = \frac{a}{o}$	$\cot x = \frac{1}{\tan x}$

For example, in the figure above, the cosine of x is the side adjacent to x (labeled a), over the hypotenuse (labeled h):

$$\cos x = \frac{a}{h}$$

If a=12cm, and h=24cm, then cos x = 0.5 (12 over 24).

Trigonometric Functions in terms of Sine and Cosine Functions

Let's use sine and cosine functions to determine the other trigonometric functions.

- cosec $x = 1/\sin x$, where $x \neq n\pi$
- sec $x = 1/\cos x$ where $x \neq (2n+1)\pi/2$
- tan $x = \sin x/\cos x$, where $x \neq (2n+1)\pi/2$
- cot $x = \cos x/\sin x$, where $x \neq n\pi$

In all the above functions, n is an integer. For all the real values of x, we already know that,

- $\sin^2 x + \cos^2 x = 1$,

This lets us know that,

- $1 + \tan^2 x = \sec^2 x$
  $1 + \cot^2 x = \text{cosec}^2 x$

We already know the values of trigonometric ratios for the angles of 0°, 30°, 45°,60° and 90°. We use the same values for trigonometric functions as well. The values of trigonometric functions thus are as shown in the table below:

For knowing the values of cosec x, sec x and cot x we reciprocate the values of sin x, cos x and tan x, respectively. From the above discussion, we can now calculate the values of the various trigonometric functions by using the respective trigonometric ratios, as stated in the table above.

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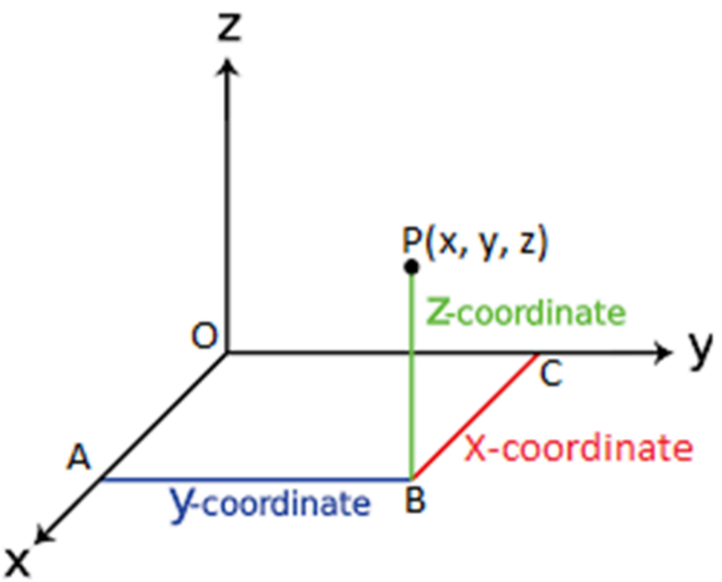
B.C.A study

Unit-5: 3 D Co-ordinate Geometry

A three-dimensional Cartesian **coordinate system** is formed by a point called the origin (denoted by O) and a basis consisting of three mutually perpendicular vectors. … The **coordinates** of any point in space are determined by three real numbers: x, y, z.

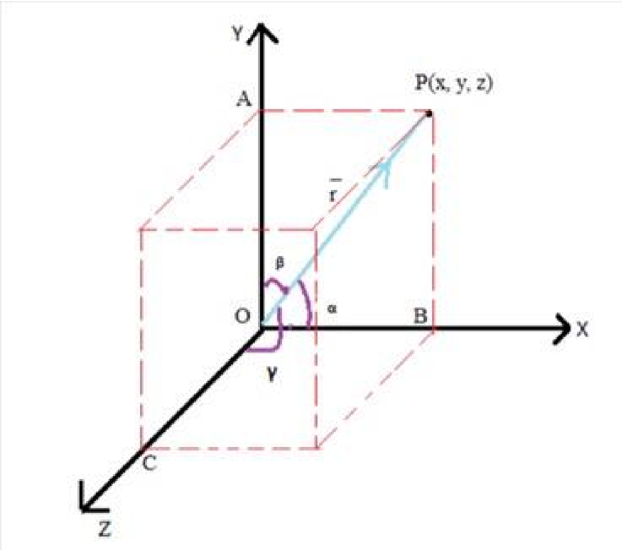
Coordinates in Space

Let there be a point P in space as shown in the figure below. If we drop a perpendicular PB on the XY plane and then from point B, we drop perpendiculars BA and BC on the x-axis and y-axis respectively. Assuming the length of the perpendiculars BC, BA and PB as x, y and z respectively. These lengths x, y and z are known as the co-ordinates of the point P in three-dimensional space. It must be noted that while giving the coordinates of a point, we always write them in order such that the co-ordinate of x-axis comes first, followed by the co-ordinate of the y-axis and the z-axis. Thus for each point in space there exist an ordered 3-tuple of real numbers for its representation.



direcion angle and direction cosine

the **direction cosines** (or **directional cosines**) of a vector are the cosines of the angles between the vector and the three coordinate axes. Equivalently, they are the contributions of each component of the basis to a unit vector in that direction.



Given a vector (a,b,c) in three-space, the **direction cosines** of this vector are

$$\cos\alpha = \frac{a}{\sqrt{a^2+b^2+c^2}},$$
$$\cos\beta = \frac{b}{\sqrt{a^2+b^2+c^2}},$$
$$\cos\gamma = \frac{c}{\sqrt{a^2+b^2+c^2}}.$$

(1)

Here the **direction angles** α, β, γ are the angles that the vector makes with the positive x-, y– and z-axes, respectively. . We have

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$$

distance between two points

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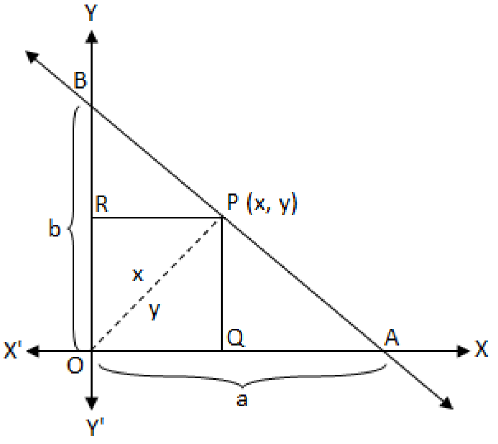
Equation of line

How do you find a standard form equation for the line with x ...

Let us see see, how to derive the slope-intercept form equation of a straight through the following steps.



Step 1 :

Let L be a line with slope m and y-intercept b. Circle the point that must be on the line. Justify your choice.

(b, 0) (0, b) (0, m) (m, 0)

The coordinate of x is 0 in the point that includes the y-intercept.

Step 2 :

Recall that slope is the ratio of change in y to change in x. Complete the equation for the slope m of the line using the y-intercept (0, b), and another point (x, y) on the line.

Slope m = change in y-values / change in x-values

Slope m = $(y - b) / (x - 0)$

Slope m = $(y - b) / x$

Step 3 :

In an equation of a line, we often want y by itself on one side of the equation. Solve the equation from Step 2 for y.

$m = (y - b) / x$

Multiply both sides by x

$m.x = [(y - b) / x].x$

$mx = y - b$

Add b to both sides of the equation.

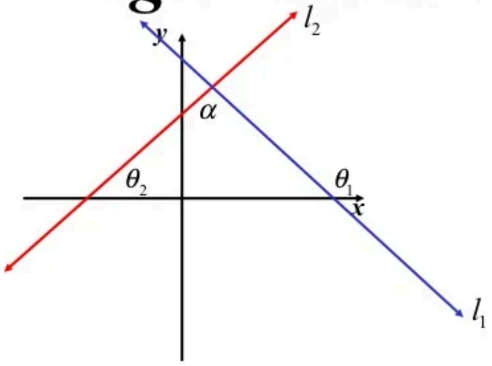
$mx + b = (y - b) + b$

$mx + b = y$

Write the equation with y on the left side.

$y = mx + b$

Angle Between Two Lines



l_1 has slope $m_1 = \tan\theta_1$

l_2 has slope $m_2 = \tan\theta_2$

$\theta_1 = \alpha + \theta_2$ (exterior $\angle\ \Delta$)

$$\alpha = \theta_1 - \theta_2$$

$$\tan\alpha = \tan(\theta_1 - \theta_2)$$

$$= \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1 \tan\theta_2}$$

$$= \frac{m_1 - m_2}{1 + m_1 m_2}$$

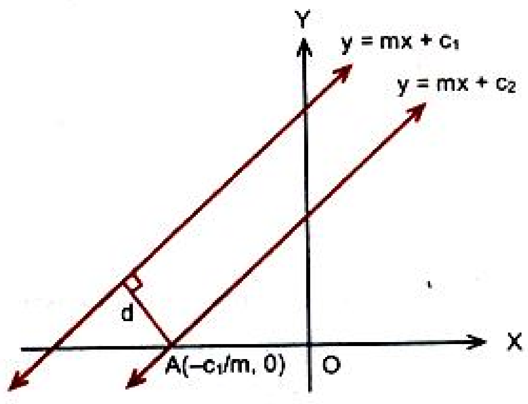
The acute angle between two lines with slopes m_1 and m_2 can be found using;

$$\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Shortest Distance Between Two Parallel Lines

Formula to find distance between two parallel line:

Consider two parallel lines are represented in the following form :



$y = mx + c_1$ …(i)

$y = mx + c_2$ ….(ii)

Where m = slope of line

Then, the formula for shortest distance can be written as under :

$$d = \frac{|c_2 - c_1|}{\sqrt{1+m^2}}$$

If the equations of two parallel lines are expressed in the following way :

$ax + by + d_1 = 0$

$ax + by + d_2 = 0$

then there is a little change in the formula.

$$d = \frac{|d_2 - d_1|}{\sqrt{a^2+b^2}}$$

shortest distance between two skew lines

Skew Lines are basically, lines that neither intersect each other nor are they parallel to each other in the three-dimensional space.The shortest distance between skew lines is equal to the length of the perpendicular between the two lines

Note that in case the two skew lines are intersecting, the shortest distance between them must necessarily be zero

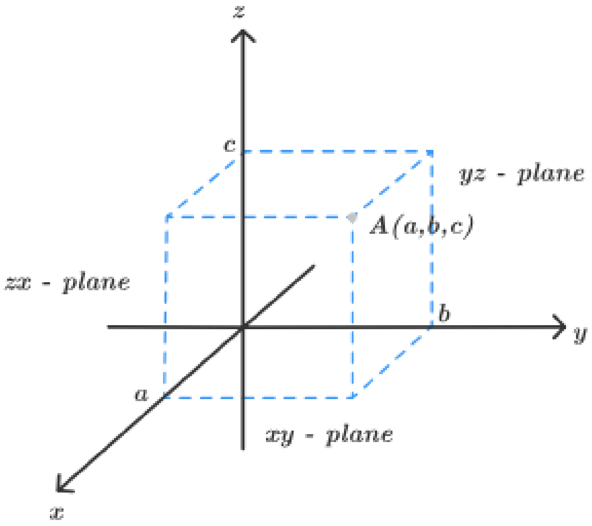
14. Shortest distance between two skew lines is the length of the line segment, which is perpendicular to the two given lines. If two given lines are $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$, then shortest distance is

$$\left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|.$$

If shortest distance is zero, then lines intersect and lines intersect in space if they are coplanar. Hence above lines are coplanar if $(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = 0$.

Plane

A **plane** in space is **defined** by three points (which don't all lie on the same line) or by a point and a normal vector to the **plane**.



Equation of plane

The equation of a plane in 3D space is defined with normal vector (perpendicular to the plane) and a known point on the plane.

Let the normal vector of a plane, $\vec{n}$ and the known point on the plane, P_1. And, let any point on the plane as P.

We can define a vector connecting from P_1 to P, which is lying on the plane.
$\vec{p} - \vec{p}_1 = (x - x_1,\ y - y_1,\ z - z_1)$

Since the vector $\vec{p} - \vec{p}_1$ and the normal vector $\vec{n}$ are perpendicular each other, the dot product of two vector should be 0.
$\vec{n} \cdot (\vec{p} - \vec{p}_1) = 0 \qquad (\because \vec{n} \perp (\vec{p} - \vec{p}_1))$

This dot product of the normal vector and a vector on the plane becomes the equation of the plane. By calculating the dot product, we get;

$$(a, b, c) \cdot (x - x_1, y - y_1, z - z_1) = 0$$
$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$
$$ax + by + cz - (ax_1 + by_1 + cz_1) = 0$$

If we substitute the constant terms to $d = -(ax_1 + by_1 + cz_1)$, then the plane equation becomes simpler;
$ax + by + cz + d = 0$

condition for a line to lie in a plane

We observe that a straight line will lie in a plane if every point on the line, lie in the plane and the normal to the plane is perpendicular to the line.

i) If the line lies in the plane $\cdot = d$, then $\cdot = d$ *and* $. = 0$

ii) if the line lies in the plane $Ax + By + Cz + D = 0$, then

$Ax_1 + By_1 + Cz_1 + D = 0$ and $aA + bB + cC = 0$

Example

Verify whether the line lies in the plane $5x - y + z = 8$.

Solution

Here, (x_1, y_1, z_1) = (3, 4, −3) and direction ratios of the given straight line are $(a,b,c) = (-4, -7, 12)$. Direction ratios of the normal to the given plane are (A, B, C) = (5, −1,1) .

We observe that, the given point (x_1, y_1, z_1) = (3, 4, −3) satisfies the given plane $5x - y + z = 8$

Next, $aA + bB + cC = (-4)(5) + (-7)(-1) + (12)(1) = -1 \neq 0$. So, the normal to the plane is not perpendicular to the line. Hence, the given line does not lie in the plane.

Equation of sphere

A sphere is defined as the set of all points in three-dimensional space that are located at a distance r (the "radius") from a given point (the "center").

Equation of the sphere with center at the origin (0,0,0) and radius R is given by

$$x^2 + y^2 + z^2 = R^2,$$

The Cartesian equation of a sphere centered at the point (x_0, y_0, z_0) with radius R is given by

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = R^2.$$

The Cartesian equation of a sphere centered at the point (x0,y0,z0) with radius R is given by.

Equation of a Tangent Plane to a Sphere Given Point of Tangency

Since the tangent plane is perpendicular to the sphere's radius to the point of tangency, the radius vector serves as the **normal** for the tangent plane. Once we know the point of contact and the coordinates of the sphere's center, we have the normal vector and a point on the plane so we can find it's equation.

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example 4: Write the equation of the tangent plane to the sphere
with equation $(x + 1)^2 + (y - 4)^2 + (z + 6)^2 = 16$ at the point P (- 4, 4, - 10).

solution: The center C is (*-1, 4, -6*). The radius vector or normal is CP = $\begin{pmatrix} -3 \\ 0 \\ -4 \end{pmatrix}$.

A vector in the plane we seek is **v** = $\begin{pmatrix} x+4 \\ y-4 \\ z+10 \end{pmatrix}$. Since the normal is z plane, **n $ v = 0**.

So, $$\begin{pmatrix} -3 \\ 0 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} x+4 \\ y-4 \\ z+10 \end{pmatrix} = 0 \Rightarrow -3(x+4) - 4(z+10) = 0$$

The equation of the tangent plane is $-3x - 4z - 52 = 0$.

Therefore, to find the equation of the tangent plane to a given sphere, dot the radius vector with any vector in the plane, set it equal to zero. We simply write the equation of the plane through point P, with normal vector equal to the vector joining the center of the sphere to the point of tangency.

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